Combined Trellis Coding and Feedforward Processing for MSS Applications

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ABSTRACT

The idea of using a multiple (more than two) symbol observation interval to improve error probability performance is applied to differential detection of trellis coded MPSK over a mobile satellite (fading) channel. Results are obtained via computer simulation. It is shown that only a slight increase (e.g., one symbol) in the length of the observation interval will provide a significant improvement in bit error probability performance both in AWGN and fading environments.

1.0 INTRODUCTION

In a previous paper [1], the notion of using a multiple symbol observation interval for differentially detecting uncoded multiple phase-shift-keying (MPSK) was introduced. In particular, the technique made use of maximum-likelihood sequence estimation of N-1 (N > 2) phases rather than symbol-by-symbol detection as in conventional (N = 2) differential detection. The amount of improvement gained over conventional differential detection was shown to be a function of the number of phases, M, and the number of additional symbols (N-2) added to the observation. Furthermore, as the number of symbols, N, in the observation interval theoretically approached infinity, the performance was shown to be identical to that corresponding to ideal *coherent* detection.

In [2], this idea was extended to trellis coded modulations (TCM), in particular, MPSK. There, it was shown that a combination of a multiple trellis coded modulation (MTCM) [3] with multiplicity (number of trellis code output symbols per input symbol) equal to N-1 combined with a multiple symbol differential detection scheme analogous to that in [1] can potentially yield a significant improvement in performance, even for small N, over that corresponding to conventional trellis coded multilevel DPSK (MDPSK).

In this paper, we give further evidence of the gain demonstrated in [2] as well as extending these notions to fading channels.

2.0 SYSTEM MODEL

Figure 1 is a simplified block diagram of the system under investigation. Input bits occurring at a rate R_b are passed through a rate nk/(n+1)k multiple trellis encoder (k is the multiplicity of the code) producing an encoded bit stream at a rate $R_s = [(n+1)k/nk]R_b$. Next, the encoded bits are divided into k groups of n+1 bits each and each

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group is mapped into a symbol selected from an $M = 2^{n+1}$ - level PSK signal set according to a set partitioning method for multiple trellis codes [3] analogous to that proposed by Ungerboeck [4] for conventional (unit multiplicity) codes. Since the MDPSK symbol rate is R_b/n, it is reasonable, from a conservation of bandwidth standpoint, to compare the performance of this system to an uncoded $M = 2^n$ level DPSK system with the identical input bit rate.

At the receiver, the noise-corrupted signal is differentially detected and the resulting symbols are then fed to the trellis decoder which is implemented with the Viterbi algorithm. In selecting a decoding metric, a tradeoff exists between simplicity of implementation and the optimality associated with the degree to which the metric matches the differential detector output statistics.

For the case of uncoded MDPSK, a metric based on minimizing the distance between the received and transmitted signal vectors is optimum in the sense of a minimum probability of error test. The specific forms of this metric for conventional and multiple differential detection were described in [1]. For conventional trellis-coded MDPSK, the metric takes on the form of a minimum squared-Euclidean distance metric. For multiple symbol detection of MTCM, the form of the distance metric is quite different. Nevertheless, as was shown in [5], by a suitable modification of the multiple trellis code design, the appropriate distance metric can be converted once again into a squared-Euclidean distance metric. The so-called "equivalent" multiple trellis code that results from this modification then becomes the key tool used for analyzing the performance of the system.

3.0 CHANNEL MODEL

The mobile satellite fading model is shown in Figure 2. A complex representation of the mathematical model for the fading channel, used to assess the performance of MTCM systems, is also illustrated in Figure 2. In this figure, F(t) represents the fading process, m(t) represents the lognormal shadowing process, N(t) is a complex Gaussian noise process, and $\omega_d = 2\pi f_d$ is the Doppler spread in units of rad/sec.

In this paper, the effect of shadowing is not accounted for. Furthermore, it is assumed that the fading is slowly varying with a normalized amplitude $\rho = |F(t)|$ which is Rician distributed, i.e.,

$$p(\rho) = 2\rho(1+K)\exp[-K-\rho^2(1+K)] \times I_0\left(2\rho\sqrt{K(1+K)}\right); \quad \rho \geq 0 \qquad (1)$$

where the parameter K is the ratio of the power in the coherent (line-of-sight and specular) component to that in the noncoherent (diffuse) component. A special case of the Rician fading model is the Rayleigh fading channel (corresponding to $K = 0$) which characterizes terrestrial mobile radio systems.

4.0 ANALYSIS MODEL

We denote a coded symbol sequence of length N_s by

$$\underline{x} = (x_1, x_2, \cdots, x_{N_s}) \qquad (2)$$

where the kth element [2] of $\underline{x}$, namely, x_k, represents the transmitted MPSK symbol in the kth transmission interval and, in general, is a nonlinear function of the state of the encoder and the nk information bits at its input. Before transmission over the channel, the sequence $\underline{x}$ is differentially encoded producing the sequence $\underline{s}$. In

[2] Here k is used as an index. As already mentioned, we shall also use k to denote the multiplicity of the code. The particular case in point should be clear from the context of the usage.

phasor notation, s_k and s_{k+1} can be written as

$$s_k = \sqrt{2P}\, e^{j\phi_k}$$

$$s_{k+1} = s_k x_{k+1} = \sqrt{2P}\, e^{j(\phi_k + \Delta\phi_{k+1})} = \sqrt{2P}\, e^{j\phi_{k+1}} \tag{3}$$

where $E_S = rE_b$ is the energy per MDPSK symbol and

$$x_k = e^{j\Delta\phi_k} \tag{4}$$

is the phasor representation of the MPSK symbol $\Delta\phi_k$ assigned by the mapper in the kth transmission interval.

The corresponding received signal in the kth transmission interval is

$$r_k = F_k s_k e^{j\theta_k} + n_k \tag{5}$$

where n_k is a sample of zero mean complex Gaussian noise with variance

$$\sigma_n^2 = \frac{2N_0}{T} \tag{6}$$

and θ_k is an arbitrary phase introduced by the channel which, in the absence of any side information, is assumed to be uniformly distributed in the interval $(-\pi,\pi)$. F_k is a sample of a complex Gaussian fading process and thus $|F_k|$ is Rician distributed.

In [1] it was shown that for uncoded MPSK the maximum-likelihood decision statistic based on an observation of N successive MPSK symbols (the present one, i.e., the kth, and N-1 in the past) is

$$\eta = \left| r_{k-N+1} + \sum_{n=0}^{N-2} r_{k-n} e^{-j\sum_{m=0}^{N-n-2} \Delta\phi_{k-n-m}} \right|^{\ell} \tag{7}$$

with $\ell = 1$ or 2. If (7) is used as a branch metric for the coded case then, for low SNR, $\ell = 2$ whereas, for high SNR, $\ell = 1$. Since the first phase in this sequence acts as the reference phase, this statistic allows a simultaneous decision to be made on N-1 phases in accordance with the particular data phase sequence $\Delta\phi_{k-N-2}, \Delta\phi_{k-N-1}, \ldots, \Delta\phi_k$ that maximizes η.

To apply the notion of multiple symbol differential detection to trellis coded MPSK, the decision statistic of (7) must be associated with a *branch* in the trellis diagram. To do this, we construct a multiple trellis code of multiplicity k = N-1. Thus, we can envision the transmitted sequence, $\underline{x}$, of (2) as being partitioned into $B = N_s/k = N_s/(N-1)$ subsequences[3], i.e.,

$$\underline{x} = (\underline{x}^{(1)}, \underline{x}^{(2)}, \ldots, \underline{x}^{(B)}) \tag{8}$$

with each subsequence $\underline{x}^{(i)} = (x_{i1}, x_{i2}, \ldots, x_{ik})$ representing an assignment to a trellis branch. Similarly, a received sequence, $\underline{r}$, of length N_s is associated with a path of length B branches in the trellis diagram. Once this association is made, computation of bit error probability for the system follows along the lines of the approach taken in [3]. The details of the analysis are presented in [2].

The total metric proposed for the multiple trellis decoder with channel state information (CSI) is[4]

$$\eta = \sum_i \left|F^{(i)}\right| \left| r^{(i)}_{k-N+1} + \sum_{n=0}^{N-2} r^{(i)}_{k-n} e^{-j\sum_{m=0}^{N-n-2} \Delta\phi^{(i)}_{k-n-m}} \right|^{\ell} \tag{9}$$

5.0 EXAMPLE - 16-STATE TRELLIS CODE

Consider a 16 state, rate 2/3 trellis coded 8PSK using conventional (N = 2) and multiple (N = 3) symbol differential detection. This code, which is optimum on the AWGN, has the transition matrix [5; Fig. 7]

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[3] Since N_s is arbitrary, we can choose it such that $N_s/(N-1)$ is integer.

[4] Note that in (9), the subscript "k" on F_k has been omitted since we assume that the fading amplitude is constant along any given branch consistingf of N - 1 symbols.

$$
T = \begin{bmatrix}
0&4&2&6&&&&&&&&&&&& \\
&&&&1&5&3&7&&&&&&&& \\
&&&&&&&&4&0&6&2&&&& \\
&&&&&&&&&&&&5&1&7&3 \\
2&6&0&4&&&&&&&&&&&& \\
&&&&3&7&1&5&&&&&&&& \\
&&&&&&&&6&2&4&0&&&& \\
&&&&&&&&&&&&7&3&5&1 \\
4&0&6&2&&&&&&&&&&&& \\
&&&&5&1&7&3&&&&&&&& \\
&&&&&&&&0&4&2&6&&&& \\
&&&&&&&&&&&&1&5&3&7 \\
6&2&4&0&&&&&&&&&&&& \\
&&&&7&3&5&1&&&&&&&& \\
&&&&&&&&2&6&0&4&&&& \\
&&&&&&&&&&&&3&7&1&5
\end{bmatrix} \qquad (10)
$$

The output of the encoder enters the block interleaver. We use 128 symbol interleaving with depth of 8 and span of 8 symbol pairs (i.e., symbol pair by symbol pair interleaving). The output symbols of the interleaver are differentially encoded after which they enter the 8PSK modulator with 100% excess bandwidth root raised cosine pulse shaping. The received signal enters the demodulator with Doppler correction which is also shown in Figure 1. Every 2T-sec., three samples r_k, r_{k-1}, r_{k-2} enter the triple symbol by triple symbol deinterleaver. The output of the deinterleaver enters the multiple (double) trellis decoder whose metric is computed based on (9). The decoder structure is based on the Cartesian product **TxT** [2].

The system in Figure 1 for the 16 state code with multiplicity two has been simulated over an additive white Gaussian noise channel with and without the inclusion of Rician fading. We assume a Rician parameter K = 10 and Doppler spreads of 20 Hz and 104 Hz. The metric used in the simulations was that given in (9) with $\ell = 2$ and no CSI, i.e., $|F^{(i)}| = 1$ for all i. The results of the simulations are shown in Figures 3 and 4 for unfaded and faded cases, respectively. We observe from these figures that in going from N = 2 to N = 3, an improvement in E_b/N_0 performance of at least 0.75 dB is obtained for the range of bit error probabilities illustrated.

6.0 CONCLUSIONS

As was true for the uncoded case, the use of multiple symbol differential detection of trellis coded MPSK can offer an improvement in error probability performance over conventional (two-symbol observation) differential detection of the same coded modulation. Again only a slight increase in the length of the observation interval is necessary to demonstrate a significant improvement both for faded and unfaded channels.

7.0 REFERENCES

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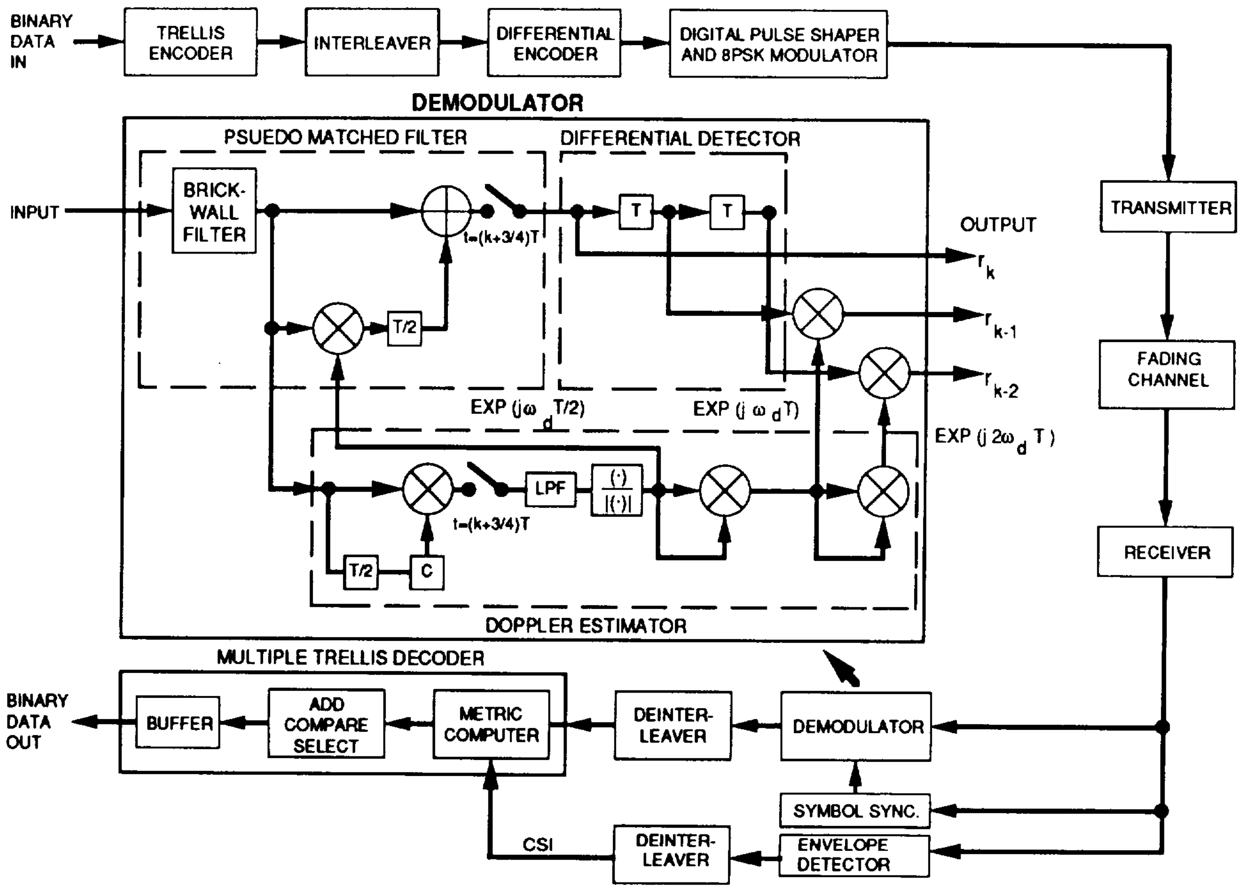

Figure 1. System Block Diagram.

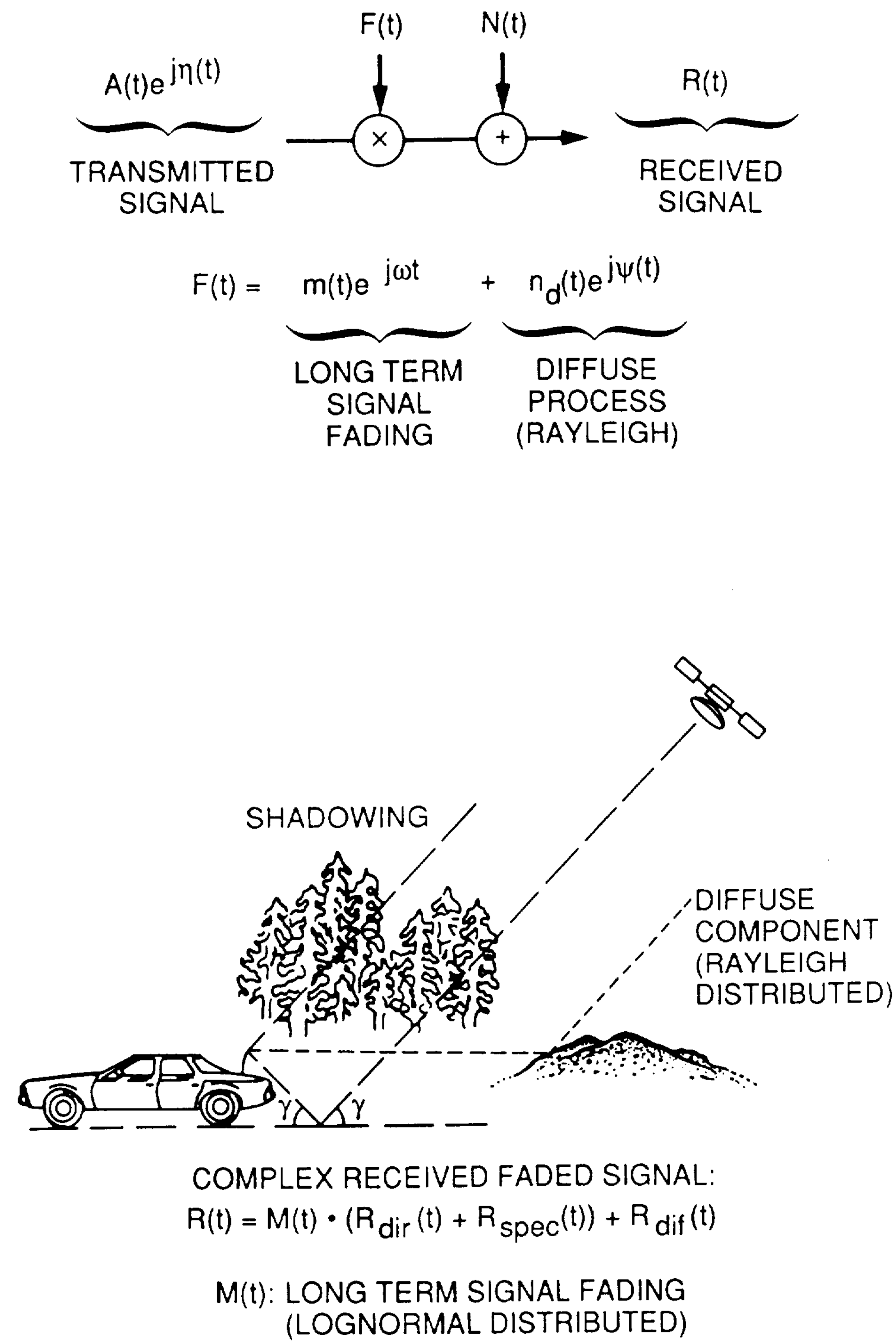

Figure 2. Mobile Satellite Channel Propagation Model.

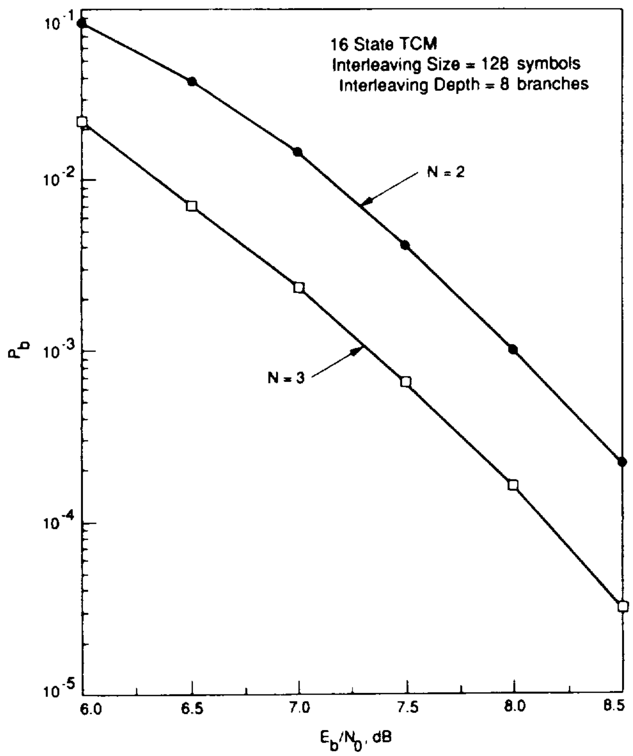

Figure 3. Simulation Results for Bit Error Probability of 16 State, Rate 2/3 Trellis Coded 8PSK With Conventional (N = 2) and Multiple (N = 3) Symbol Differential Detection (No Fading).

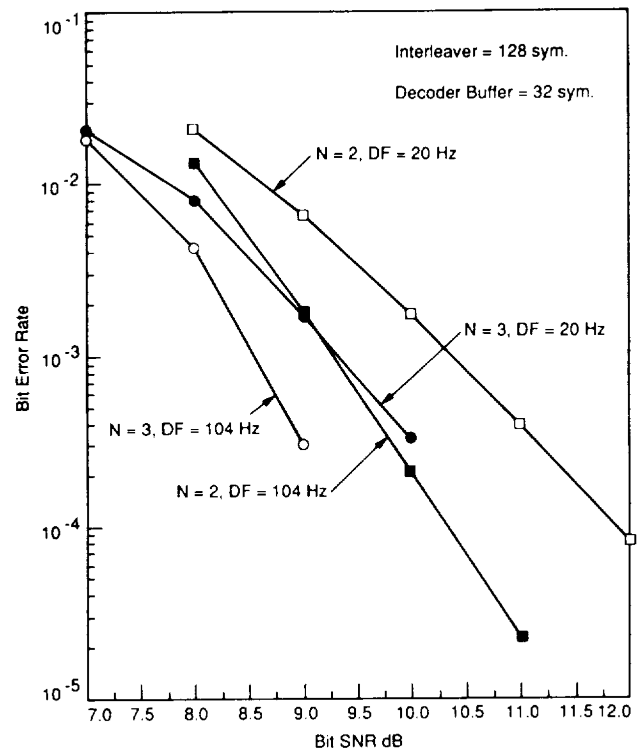

Figure 4. Simulation Results for Bit Error Probability of 16 State, Rate 2/3 Trellis Coded 8PSK With Conventional (N = 2) and Multiple (N = 3) Symbol Differential Detection (Rician Fading With K = 10).